# Geometry: 2.1-2.3 Notes

**NAME**______________________

**2.1 Be able to write all types of conditional statements.**__________________ **Date:**___________________

**Define Vocabulary:**

conditional statement

if-then form

hypothesis

conclusion

negation

converse

inverse

contrapositive

equivalent statements

perpendicular lines

biconditional statement

truth value

truth table

## Conditional Statement

A **conditional statement** is a logical statement that has two parts, a *hypothesis p* and a *conclusion q*. When a conditional statement is written in **if-then form**, the "if" part contains the **hypothesis** and the "then" part contains the **conclusion**.

**Words** If $p$, then $q$. **Symbols** $p \rightarrow q$ (read as "$p$ implies $q$")

**Examples: Use (H) to identify the hypothesis and (C) to identify the conclusion. Then write each conditional in if-then form.**

**WE DO**

**1a.** $x > 5$ if $x > 3$.

**1b.** All members of the soccer team have practice today.

**YOU DO**

**2a.** All 30° angles are acute angles.

**2b.** $2x + 7 = 1$, because $x = -3$.

## Negation

The **negation** of a statement is the *opposite* of the original statement. To write the negation of a statement $p$, you write the symbol for negation $(\sim)$ before the letter. So, "not $p$" is written $\sim p$.

**Words** not $p$ **Symbols** $\sim p$

**Examples: Write the negation of each statement.**

**WE DO**

1a. The car is white.

1b. It is not snowing.

**YOU DO**

2a. The shirt is green.

2b. The shoes are not red.

## Related Conditionals

Consider the conditional statement below.

**Words** If $p$, then $q$. **Symbols** $p \rightarrow q$

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**Converse** To write the **converse** of a conditional statement, exchange the hypothesis and the conclusion.

**Words** If $q$, then $p$. **Symbols** $q \rightarrow p$

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**Inverse** To write the **inverse** of a conditional statement, negate both the hypothesis and the conclusion.

**Words** If not $p$, then not $q$. **Symbols** $\sim p \rightarrow \sim q$

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**Contrapositive** To write the **contrapositive** of a conditional statement, first write the converse. Then negate both the hypothesis and the conclusion.

**Words** If not $q$, then not $p$. **Symbols** $\sim q \rightarrow \sim p$

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A conditional statement and its contrapositive are either both true or both false. Similarly, the converse and inverse of a conditional statement are either both true or both false. In general, when two statements are both true or both false, they are called **equivalent statements**.

**Examples: Write each statement in words and then decide whether it is true or false.**

**WE DO**

Let $p$ be "you are in New York City" and let $q$ be "you are in the United States."

**a.** the conditional statement $p \rightarrow q$

**b.** the converse $q \rightarrow p$

**c.** the inverse $\sim p \rightarrow \sim q$

**d.** the contrapositive $\sim q \rightarrow \sim p$

**YOU DO**

Let $p$ be "the stars are visible" and let $q$ be "it is night."

**a.** the conditional statement $p \rightarrow q$

**b.** the converse $q \rightarrow p$

**c.** the inverse $\sim p \rightarrow \sim q$

**d.** the contrapositive $\sim q \rightarrow \sim p$

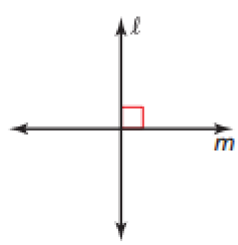
## Using Definitions

You can write a definition as a conditional statement in if-then form or as its converse. Both the conditional statement and its converse are true for definitions. For example, consider the definition of *perpendicular lines*.

If two lines intersect to form a right angle, then they are **perpendicular lines**.

You can also write the definition using the converse: If two lines are perpendicular lines, then they intersect to form a right angle.

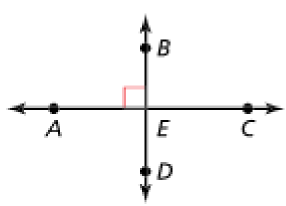
You can write "line $\ell$ is perpendicular to line $m$" as $\ell \perp m$.




$\ell \perp m$

**Examples: Use the diagrams. Decide whether the statement is true. Explain your answer using the definitions you have learned.**

**WE DO**

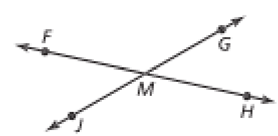



**a.** $m\angle AEB = 90°$

**b.** *Points A, C,* and *D* are collinear.

**c.** $\overrightarrow{AC}$ and $\overrightarrow{CA}$ are opposite rays.

**YOU DO**




**6.** $\angle JMF$ and $\angle FMG$ are supplementary.

**7.** Point $M$ is the midpoint of $\overline{FH}$.

**8.** $\angle JMF$ and $\angle HMG$ are vertical angles.

## Biconditional Statement

When a conditional statement and its converse are both true, you can write them as a single *biconditional statement*. A **biconditional statement** is a statement that contains the phrase "if and only if."

**Words** $p$ if and only if $q$ **Symbols** $p \leftrightarrow q$

Any definition can be written as a biconditional statement.

**Examples: Rewrite the definition as a single biconditional statement.**

**<u>WE DO</u>**

If two angles are complementary, then the sum of the measure of the angles is 90 degrees.

**<u>YOU DO</u>**

If two line segments have the same length, then They are congruent segments.

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| Assignment | |

**2.2 Use inductive and deductive reasoning.____________________________ Date:____________________**

**Define Vocabulary:**

conjecture

inductive reasoning

counterexample

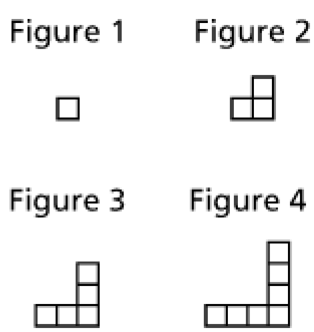
deductive reasoning

## Inductive Reasoning

A **conjecture** is an unproven statement that is based on observations. You use **inductive reasoning** when you find a pattern in specific cases and then write a conjecture for the general case.

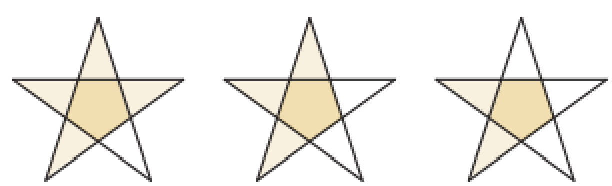
**Examples: Sketch the next figure.**

**WE DO**

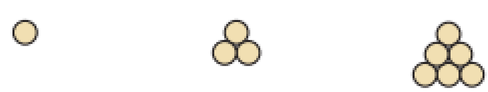



**YOU DO**

**a.**



**b.**



**Examples:** Make and test a conjecture.

**WE DO**

**The product of a negative integer and a positive integer.**

**YOU DO**

**The sum of any five consecutive integers.**

## Counterexample

To show that a conjecture is true, you must show that it is true for all cases. You can show that a conjecture is false, however, by finding just one *counterexample*. A **counterexample** is a specific case for which the conjecture is false.

**Examples: Find a counter example to disprove the conjecture.**

**WE DO**

**The absolute value of the sum of the two numbers is equal to the sum of the two numbers.**

**YOU DO**

**The sum of two numbers is always greater than their difference.**

## Deductive Reasoning

**Deductive reasoning** uses facts, definitions, accepted properties, and the laws of logic to form a logical argument. This is different from *inductive reasoning*, which uses specific examples and patterns to form a conjecture.

## Laws of Logic

### Law of Detachment

If the hypothesis of a true conditional statement is true, then the conclusion is also true.

**Statement**: If I save up money, I can buy a new horse

**You're told**: Joe saved up money. ***You can conclude:*** Joe can buy a new horse.

**You're told**: Ms. K bought a new horse. ***You cannot make a conclusion***. *The information doesn't match up with the original hypothesis.*

### Law of Syllogism

If hypothesis $p$, then conclusion $q$.
If hypothesis $q$, then conclusion $r$. — If these statements are true,

If hypothesis $p$, then conclusion $r$. — then this statement is true.

**Examples: Use the law of detachment.**

**<u>WE DO</u>**

If a figure is a square, then it is a rectangle. You know that quadrilateral *ABCD* is a square. Using the Law of Detachment, what statement can you make?

**<u>YOU DO</u>**

If $90° < m\angle R < 180°$, then $\angle R$ is obtuse. The measure of $\angle R$ is $155°$. Using the Law of Detachment, what statement can you make?

**Examples: If possible use the law of syllogism to write a new conditional statement.**

**WE DO**

a. If soccer practice is cancelled, then you can go to the mall after school. If it is raining today, then soccer practice is cancelled.

b. If a figure is a rectangle, then all interior angles are right angles. If a figure is a square, then all interior angles are right angles.

**YOU DO**

If you get an A on your math test, then you can go to the movies.

If you go to the movies, then you can watch your favorite actor.

**Examples: Comparing Inductive and Deductive Reasoning**

Decide whether inductive or deductive reasoning is used. Explain your answer.

**WE DO**

a. If the sum of the digits of a number is divisible by 3, then the number is divisible by 3. The sum of the digits of the number 147 is 12. So, the number 147 is divisible by 3.

b. Each time you forget to do your math homework, your parents take away your phone privileges for a day. So, the next time your forget to do your math homework, you will lose your phone privileges.

**YOU DO**

All multiples of 8 are divisible by 4.
64 is a multiple of 8.
So, 64 is divisible by 4.

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| Assignment | |

**2.3 Identify postulates using diagrams._________________________________ Date:________________________**

**Define Vocabulary:**

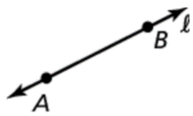
line perpendicular to a plane

# Postulates

## Point, Line, and Plane Postulates

### Postulate / Example

**2.1 Two Point Postulate**
Through any two points, there exists exactly one line.




Through points *A* and *B*, there is exactly one line ℓ. Line ℓ contains at least two points.

**2.2 Line-Point Postulate**
A line contains at least two points.

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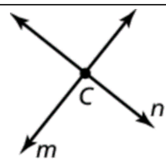
**2.3 Line Intersection Postulate**
If two lines intersect, then their intersection is exactly one point.




The intersection of line *m* and line *n* is point *C*.

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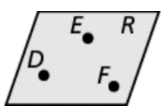
**2.4 Three Point Postulate**
Through any three noncollinear points, there exists exactly one plane.




Through points *D*, *E*, and *F*, there is exactly one plane, plane *R*. Plane *R* contains at least three noncollinear points.

**2.5 Plane-Point Postulate**
A plane contains at least three noncollinear points

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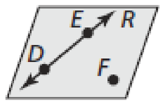
**2.6 Plane-Line Postulate**
If two points lie in a plane, then the line containing them lies in the plane.




Points *D* and *E* lie in plane *R*, so $\overleftrightarrow{DE}$ lies in plane *R*.

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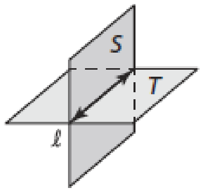
**2.7 Plane Intersection Postulate**
If two planes intersect, then their intersection is a line.




The intersection of plane *S* and plane *T* is line ℓ.

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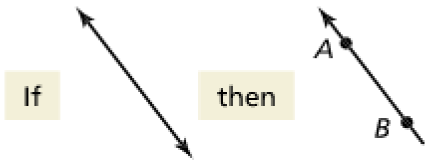
**Examples: Identify a postulate using a diagram.**

**<u>WE DO</u>**

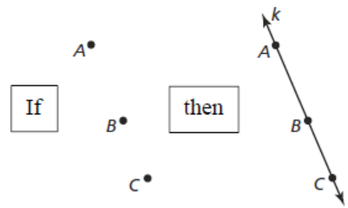
**<u>YOU DO</u>**

a.

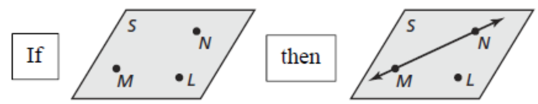



b.


1.



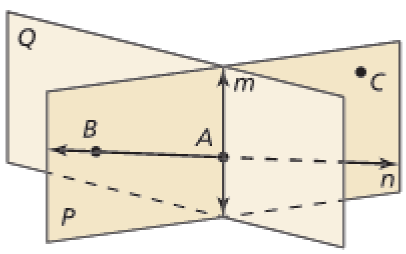

2.




**Examples: Identifying postulates from diagrams.**

**<u>WE DO</u>**

1. Write an example of the three points postulate.




2. Write an example of the two point postulate.

**<u>YOU DO</u>**

3. Write an example of the line – point postulate.

4. Write an example of the line intersection postulate.

**Examples: Sketching a diagram**

**<u>WE DO</u>**

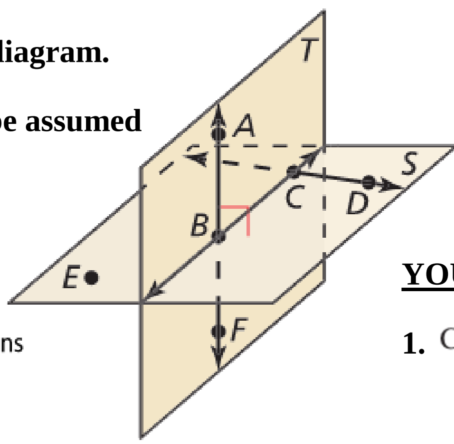
**Show $\overrightarrow{VX}$ intersecting $\overleftrightarrow{UW}$ at V so that $\overrightarrow{VX}$ is perpendicular to $\overleftrightarrow{UW}$ and U, V, and W are collinear.**

**<u>YOU DO</u>**

**$\overrightarrow{RS}$ bisecting $\overline{KL}$ at point R.**

**Examples: Interpreting a diagram.**

**Which statements *cannot* be assumed from the diagram?**




**<u>WE DO</u>**

**1.** There exists a plane that contains points *A*, *D*, and *E*.

**2.** $AB = BF$.

**<u>YOU DO</u>**

**1.** Can you assume that plane $S$ intersects plane $T$ at $\overleftrightarrow{BC}$?

**2.** Explain how you know that $\overleftrightarrow{AB} \perp \overleftrightarrow{BC}$.

| Assignment | |
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